BOSWELL-BÈTA

James Boswell Exam
VWO Mathematics A – Practice exam 2
Solution key

Date:	
Time:	3 hours
Number of questions:	6
Number of subquestions:	24
Number of supplements:	1
Total score:	72

Subject-specific marking rules and guidelines

1. For each error or mistake in calculation a single point will be subtracted from the maximum score that can be obtained for that particular part of the question.
2. If a required explanation, deduction or calculation has been omitted or has been stated incorrectly 0 points will be awarded, unless otherwise stated in the solution key. This is also the case for answers obtained by the use of a graphic calculator. Answers obtained by the graphic calculator should indicate how the graphic calculator has been used to obtain the answer. Candidates must make sure they mention formulas applied or provide lists and calculation methods used in their answers.
3. If a notational error has been made, but the error can be seen to have no influence on the final result, no points will be deducted from the total score. If, however, it is not possible to determine that there is no influence on the final result a point will be deducted from the final score.
4. A particular mistake in the answer to a particular exam question will lead to a deduction of points only once, unless the question is substantially simplified by the mistake and/or when the solution key specifies otherwise.
5. A repeated mistake made in the answer to different exam questions will lead to a deduction of points each time such a mistake has been made, unless the solution key specifies otherwise.
6. If only one example, reason, explication, explanation or any other type of answer is required and more than one has been given, only the first answer given will be graded. If more than one example, reason, explication, explanation or any other type of answer is required, only the first answers are graded, up to and including the number of answers specified by the exam question.
7. If the candidate fails to give a required unit in the answer to a question a single point will be subtracted from the total score, unless the unit has been specified in the exam question.
8. If during intermediate steps results are rounded, resulting in an answer different from one in which non-rounded intermediate results are used, one point will be subtracted from the total score. Rounded intermediate results may, however, be noted down
Exceptions to this rule are those cases in which the context of the question requires the rounding of intermediate results. The maximum number of points deducted from the total score due to rounding errors is 2 for the entire exam.

Examples for the exceptions to rule 8.
Rounding off intermediate results can be forced by the context if, for example

- The amount of money for a single good has to be rounded to two decimals;
- The number of persons, things, etc. in a concrete situation (i.e. not for an average or expected value) has to be rounded to the nearest integer.

A required level of accuracy can be forced by the context if, for example

- The answer would not be distinguishable from a trivial answer. This can occur with the rounding of growth factors or probabilities to 0 or 1. A probability of $\left(\frac{1}{6}\right)^5$ may be rounded to 0.0001 but not to 0.000.

The forced rounding up or down of answers can occur, for example

- If the exam question specifies a minimum or maximum amount. (For example, if the question is: ‘What is the minimal distance an athlete has to jump to gain a certain number of points in a contest?’)

The above examples by no means exhaust all possible cases.

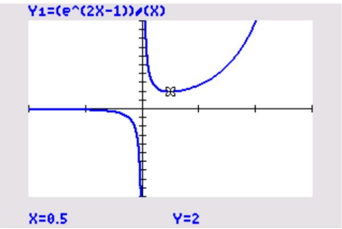
Question 1: Eggs

a	The number of company codes is $9 \cdot 10^4$	1
	The number of egg codes is $4 \cdot 25 \cdot 9 \cdot 10^4 \cdot 100$	1
	The answer: 900 000 000 egg codes	1
b	*Alternative 1:*	
	In 2017, exports were worth $\frac{409 \text{ mln}}{1.136} = 360.0 \ldots \text{mln}$ euros	1
	So exports have increased by $409 - 360.0 \ldots \approx 49$ (million euros)	1
	Alternative 2:	
	$409 \text{ mln} \cdot \frac{13.6}{113.6}$ $(= 48.9 \ldots \text{mln})$	1
	So exports have increased by approximately 49 million euros	1
c	$d = \frac{216-198}{6} = 3$	1
	Direct formula: $N_t = 3t + 198$	1
	Recursive formula: $N_t = N_{t-1} + 3$ with $N_0 = 198$	1
d	*Alternative 1:*	
	Insight that the equation $3t + 198 = 250$ has to be solved	1
	$3t = 52$ gives $t = 17\frac{1}{3}$	1
	So in the year $(2013 + 18 =)$ 2031	1
	Alternative 2:	
	Describing how the (recursive or direct) formula of N can be entered in the graphing calculator	1
	A table: t | N_t 17 | 249 18 | 252	1
	So in the year $(2013 + 18 =)$ 2031	1

Question 2: Cows

a	P(healthy and positive) $=$ P(healthy)$\cdot$ P(positive when healthy) $= \frac{3}{4} \cdot 0.02$	1
	The answer: 0.015	1
b	Insight that: P(positive) $=$ P(healthy and positive) $+$ P(sick and positive)	1
	P(sick and positive) $=$ P(sick)$\cdot$ P(positive when sick) $= \frac{1}{4} \cdot 0.85\ (= 0.2125)$	1
	$\mathrm{P}(\text{positive}) = 0.015 + 0.2125 = 0.2275$	1
c	Insight that X is binomially distributed with $n = 20$ and $p = 0.2275$	1
	$\mathrm{P}(2 < X \leq 8) = \mathrm{P}(X \leq 8) - \mathrm{P}(X \leq 2)$ $= binomcdf(20, 0.2275, 8) - binomcdf(20, 0.2275, 2)$ If a candidate uses a graphing calculator with which $\mathrm{P}(3 \leq X \leq 8)$ can be computed directly, all points can be awarded (as long as the calculation is correct)	2
	The answer: 0.843	1
d	$\mu =$ average weight of the food that a sick cows eats per day (in kg) If a candidate does not explicitly write down the meaning of μ, no points will be subtracted	
	$H_0: \mu = 70.0$	1
	$H_a: \mu < 70.0$	1
e	$\overline{Y} =$ average weight of the food that the thirty cows eat per day (in kg)	
	Insight that $\overline{Y}$ is normally distributed with $\mu = 70.0$ and $\sigma = \frac{12.5}{\sqrt{30}}$ if H_0 were true	1
	Calculating the p-value: $\mathrm{P}(\overline{Y} \leq 62.3) = normalcdf\left(-10^{99}, 62.3, 70.0, \frac{12.5}{\sqrt{30}}\right)$	1
	The p-value: 0.0004 (or more accurate)	1
	$0.0004 < 0.05$ (so H_0 is rejected and H_a has been shown)	1
	It has been shown (with $\alpha = 0.05$) that sick cows on average eat less food than healthy cows.	1

Question 3: Differentiation

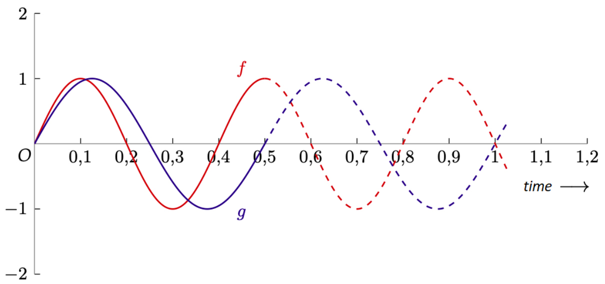
a	*Alternative 1:*	
	$f(x) = 6 \cdot \frac{1}{x^{\frac{1}{3}}} - x^1 \cdot x^{\frac{1}{2}} = 6x^{-\frac{1}{3}} - x^{1\frac{1}{2}}$	2
	$f'(x) \left(= 6 \cdot -\frac{1}{3} \cdot x^{-1\frac{1}{3}} - 1\frac{1}{2} \cdot x^{\frac{1}{2}} \right) = -2x^{-1\frac{1}{3}} - 1\frac{1}{2}x^{\frac{1}{2}}$	1
	$f'(x) = -2 \cdot \frac{1}{x^{1\frac{1}{3}}} - 1\frac{1}{2}x^{\frac{1}{2}} = -\frac{2}{x\sqrt[3]{x}} - 1\frac{1}{2}\sqrt{x}$ (or a similar expression without negative or fractional exponents)	1
	Alternative 2:	
	$f(x) = 6 \cdot \frac{1}{x^{\frac{1}{3}}} - x \cdot x^{\frac{1}{2}} = 6x^{-\frac{1}{3}} - x \cdot x^{\frac{1}{2}}$	1
	Using the product rule: $f'(x) \left(= 6 \cdot -\frac{1}{3} \cdot x^{-1\frac{1}{3}} - \left(1 \cdot x^{\frac{1}{2}} + x \cdot \frac{1}{2}x^{-\frac{1}{2}} \right) \right) = -2x^{-1\frac{1}{3}} - x^{\frac{1}{2}} - \frac{1}{2}x \cdot x^{-\frac{1}{2}}$	2
	$f'(x) = -2 \cdot \frac{1}{x^{1\frac{1}{3}}} - x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{2}} = -\frac{2}{x\sqrt[3]{x}} - 1\frac{1}{2}\sqrt{x}$ (or a similar expression without negative or fractional exponents)	1
b	$g'(x) = \frac{x \cdot 2 \cdot e^{2x-1} - e^{2x-1} \cdot 1}{x^2} \left(= \frac{2x \cdot e^{2x-1} - e^{2x-1}}{x^2} \right)$ Subtract one point if a candidate does not apply the chain rule or uses the chain rule incorrectly	3
	Insight that the equation $g'(x) = 0$ has to be solved	1
	Describing how the equation $g'(x) = 0$ can be solved • algebraically: ○ $\frac{2x \cdot e^{2x-1} - e^{2x-1}}{x^2} = 0$ ○ $2x \cdot e^{2x-1} - e^{2x-1}$ ○ $e^{2x-1} \cdot (2x - 1) = 0$ ○ $2x - 1 = 0$ ($e^{2x-1} = 0$ has no solutions) ○ $x = \frac{1}{2}$ • graphic-numerically: ○ $Y_1 = \frac{2x \cdot e^{2x-1} - e^{2x-1}}{x^2}$ (or a similar expression) ○ Option zero gives: $x = \frac{1}{2}$	1
	A plot shows that the graph of g has a minimum point at $x = \frac{1}{2}$  	
	$g\left(\frac{1}{2}\right) \left(= \frac{e^{2 \cdot \frac{1}{2} - 1}}{\frac{1}{2}} = \frac{1}{\frac{1}{2}} \right) = 2$, so the coordinates of the minimum point are $\left(\frac{1}{2}, 2\right)$	1

Question 4: Mushrooms

<table>
<tr><td>a</td><td>$X =$ diameter of a portobello (in cm)
X is normally distributed with $\mu = 11.2$ and $\sigma = 0.6$</td><td></td></tr>
<tr><td></td><td>$invNorm(0.90, 11.2, 0.6)$ $(= 11.96 \ldots)$
If a candidate uses the probability 0.10 and the option ‘right tail’, all points can be awarded</td><td>1</td></tr>
<tr><td></td><td>The answer: (at least) 12.0 (cm)</td><td>1</td></tr>
<tr><td>b</td><td>$Y =$ weight of a button mushroom (in grams)
Y is normally distributed with $\mu = 13.0$ and $\sigma = 1.9$</td><td></td></tr>
<tr><td></td><td>$S =$ total weight of n button mushrooms (in grams)</td><td></td></tr>
<tr><td></td><td>Insight that S is normally distributed with $\mu = n \cdot 13.0$ and $\sigma = \sqrt{n} \cdot 1.9$</td><td>2</td></tr>
<tr><td></td><td>Describing how the inequality $\mathrm{P}(S < 250) < 0.05$ can be solved
Alternative 1:
○ $Y_1 = normalcdf\left(-10^{99}, 250, x \cdot 13.0, \sqrt{x} \cdot 1.9\right)$
○ A table:
<table><tr><th>n</th><th>$\mathrm{P}(S < 250)$</th></tr><tr><td>20</td><td>0.120</td></tr><tr><td>21</td><td>0.004</td></tr></table>
Manier 2:
○ $Y_1 = normalcdf(-10^{99}, 250, x \cdot 13.0, \sqrt{x} \cdot 1.9)$ en $Y_2 = 0.05$
○ Option intersect gives $x = 20.3 \ldots$</td><td>1</td></tr>
<tr><td></td><td>So (at least) 21 mushrooms</td><td>1</td></tr>
<tr><td>c</td><td>On the vertical axis a logarithmic scale has been used and the graph is a (decreasing) straight line (so the number of Dutch mushroom farms decreased exponentially)</td><td>1</td></tr>
<tr><td>d</td><td>Determining the coordinates of two points, for example $(2005, 300)$ and $(2015, 100)$
The two points have to be chosen (with such accurate reading) that the half life time is between 6.15 and 6.45 years</td><td>1</td></tr>
<tr><td></td><td>The growth factor per year equals $g = \left(\frac{100}{300}\right)^{1/10} = 0.8959 \ldots$</td><td>1</td></tr>
<tr><td></td><td>Describing how the equation $0.8959 \ldots^{t} = \frac{1}{2}$ can be solved
• algebraically:
○ $0.8959 \ldots^{t} = \frac{1}{2}$
○ $t = \log_{0.8959\ldots}\left(\frac{1}{2}\right)$ $(= 6.30 \ldots)$
• graphic-numerically:
○ $Y_1 = 0.8959 \ldots^{x}$ and $Y_2 = \frac{1}{2}$
○ Option intersect gives $x = 6.30 \ldots$</td><td>1</td></tr>
<tr><td></td><td>The answer: 6 years and $(0.30 \ldots \cdot 12 \approx)$ 4 months (or 76 months)</td><td>1</td></tr>
<tr><td>e</td><td>Alternative 1:</td><td></td></tr>
<tr><td></td><td>The number of mushroom farms has become $\frac{50}{46} = 1.086 \ldots$ times as small, so the average surface area has become $1.086 \ldots$ times as large</td><td>1</td></tr>
<tr><td></td><td>On January 1st, 2020 the average surface area per farm equals $1.086 \ldots \cdot 5400 \approx 5900$ m^2 (of more accurate)</td><td>1</td></tr>
</table>

	Alternative 2:	
	Insight that the product of the number of mushroom farms and the average surface area per farm is constant	1
	On January 1st, 2020 the average surface area per farm equals $\frac{5400 \cdot 50}{46} \approx 5900$ m^2 (of more accurate)	1
	Alternative 3:	
	Suppose x is the number of farms and y is the average surface area per farm. Then $y = \frac{a}{x}$.	
	$x = 50$ and $y = 5400$ gives $a(= 50 \cdot 5400) = 270\ 000$, so $y = \frac{270\ 000}{x}$	1
	$x = 46$ gives $y = \frac{270\ 000}{46} \approx 5900$ m^2 (of more accurate)	1

Question 5: Sound

a	$I = 10^{-5}$ gives $S = 10 \cdot \log\left(\frac{10^{-5}}{10^{-12}}\right) = 70$ (dB)	1
b	$10 \cdot \log\left(\frac{2 \cdot I}{10^{-12}}\right) = 10 \cdot \log\left(\frac{I}{10^{-12}} \cdot 2\right) = 10 \cdot \left(\log\left(\frac{I}{10^{-12}}\right) + \log(2)\right)$	1
	Expanding the brackets gives: $10 \cdot \log\left(\frac{I}{10^{-12}}\right) + 10 \cdot \log(2)$	1
	$10 \cdot \log\left(\frac{I}{10^{-12}}\right) + 10 \cdot \log(2) \approx 10 \cdot \log\left(\frac{I}{10^{-12}}\right) + 3.01$, so $a \approx 3.01$	1
c	*Alternative 1:*	
	$\log\left(\frac{I}{10^{-12}}\right) = \frac{1}{10} \cdot S$	1
	$\frac{I}{10^{-12}} = 10^{\frac{1}{10} \cdot S}$	1
	$I = 10^{\frac{1}{10} \cdot S} \cdot 10^{-12}$	1
	$I = 10^{\frac{1}{10} \cdot S - 12}$, so $p = \frac{1}{10}$ and $q = 12$	1
	Alternative 2:	
	$\log\left(\frac{I}{10^{-12}}\right) = \frac{1}{10} \cdot S$	1
	$\log(I) - \log(10^{-12}) = \frac{1}{10} \cdot S$	1
	$-\log(10^{-12}) = 12$, so: $\log(I) = \frac{1}{10} \cdot S - 12$	1
	$I = 10^{\frac{1}{10} \cdot S - 12}$, so $p = \frac{1}{10}$ and $q = 12$	1
d	*Alternative 1:*	
	Draw the graphs on the supplement:  	1
	So after 1 (millisecond)	1
	Alternative 2:	
	The period of f is 0.4 and the period of g is 0.5 (milliseconds)	1
	The graphs intersect on the horizontal axis for the second time after 1 millisecond (the least common multiple of 0.2 and 0.25)	1
e	The amplitude of h is $(2 \cdot 1 =)$ 2 so $b = 2$	1
	The period of h is $\left(\frac{0.4}{2} =\right) 0.2$	1
	$c = \frac{2\pi}{\text{period}} = \frac{2\pi}{0.2} = 10\pi \ (\approx 31.4)$	1
	h reaches a maximum after a quarter of the period $t_{\text{start}} = 0 - \frac{1}{4} \cdot 0.2 = -0.05$, so $d = -0.05$ ($\pm$ a multiple of 0.2)	1

Question 6: Car manufacturing

a	$t(1) = 1000$ and $t(2)$ $(= 1000 \cdot 2^{-0.8}) = 574.34\ldots$	1
	$\left(\frac{574.34\ldots - 1000}{1000} \cdot 100\% \approx -42.6\%, \text{so}\right)$ the number of men hours decreases by 42.6%	1
b	$t'(n) = 1000 \cdot -0.8 \cdot n^{-1.8}$ $(= -800 \cdot n^{-1.8})$	1
	$n^{-1.8}$ is positive, so $t'(n) = -800 \cdot n^{-1.8}$ is negative for all values of n	1
	Because the derivative is negative, the number of men hours (t) decreases with each subsequent series (n)	1
c	$1000 \cdot n^{-0.8} = t$ gives $n^{-0.8} = \frac{1}{1000} \cdot t$	1
	$n = \left(\frac{1}{1000} \cdot t\right)^{\frac{1}{-0.8}}$	1
	$n = \left(\frac{1}{1000}\right)^{\frac{1}{-0.8}} \cdot t^{\frac{1}{-0.8}} \approx 5623 \cdot t^{-1.25}$ (of more accurate)	1